Random sequences of quantum bits

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Martin-Löf (1966) formalised the intuitive notion of randomness for infinite sequences of bits via algorithmic tests. What happens if we replace classical bits by quantum bits?

We first review the framework to formalise infinite sequences of qubits as states of a suitable C^* algebra that can be seen as coherent sequences of density matrices of dimension 2^n."Diagonal" states are equivalent to probability measures on Cantor space, i.e., statistical superpositions of classical bit sequences. The main topic of the talk is joint work with the mathematical physicist Volkher Scholz: we introduce an analog of Martin-Löf's notion for such sequences. For classical bit sequences (i.e., Dirac measures), the two notions coincide.

We discusstwo versions of quantum Kolmogorov complexity for finite sequences of qubits and their relationship to quantum Martin-Löf randomness. We discuss results of Tejas Bhojraj, a student of J. Miller, from his 2021 thesis on quantum ML-randomness. For instance, he showed that Martin-Loef and Solovay tests are equivalent in the quantum setting, and showed that incompressibility in terms of QK-complexity is equivalent to being Solovay random in a weak sense.

We also use the notion of quantum ML-randomness towards an effective version of the Shannon-McMillan-Breiman theorem in the quantum setting, in joint work in progress with Tomamichel. Recent work with Stephan addresses the easier case of measures, which is of interest by itself, and serves as a testing ground for conjectures in the more general setting.

References:

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