Π^0_1 classes relative to an enumeration oracle

Jun Le Goh

University of Wisconsin-Madison
junle.goh@wisc.edu

Since Π^0_1-classes are essential in the study of the Turing degrees, one would expect an analogous notion to be useful in the study of the enumeration degrees (e-degrees). Such a notion, that of a $\Pi^0_1\langle X\rangle$-class, was introduced by Miller and Soskova. They showed that a basic property of Π^0_1-classes fails in this setting: There are enumeration oracles which are PA relative to themselves.

We study the extent to which various other properties of the relation "PA above" hold in the e-degrees, such as the existence of a universal class. This defines several apparently new subclasses of the e-degrees. We characterize three other subclasses of e-degrees, first studied by Kalimullin and Puzarenko, by showing that they are analogs of our subclasses, obtained by considering only $\langle X\rangle$-separating classes instead of all $\Pi^0_1\langle X\rangle$-classes. My talk will focus on our construction (by a tree forcing) of an enumeration oracle which has a universal function but no universal class.

Joint work with Iskander Kalimullin, Joseph Miller and Mariya Soskova.