One point extensions of antichains in the local structure of the enumeration degrees

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We would like to characterize the complexity of the theory of various degree structures and their fragments. For the Turing degrees and its local substructure of the Δ^0_2 degrees we know exactly at what level of quantifier complexity decidability breaks down: the two quantifier theory is decidable, while the three quantifier theory is not. For the c.e. degrees, the enumeration degrees and the local structure of the Σ^0_2 enumeration degrees there is a gap in our knowledge: we know that the one quantifier theory is decidable and that the three quantifier theory is not decidable, but in each case we do not know what happens at level two.

The decidability of the two quantifier theory can be phrased structurally as follows: decide whether given any partial order P and finitely many extensions, of P, say $Q_1, Q_2, \dots Q_k$ every embedding of P in the considered degree structure extends to an embedding of at least one of the Q_i. The case when k=1 is known as the extension of embeddings problem and it is decidable for each of the structures.

In this talk I will describe our partial progress towards a renewed attack of a further sub-problem for the local substructure of the enumeration degrees: given an antichain P and finitely many one point extensions of P, say $Q_1, Q_2, \dots Q_k$, placing a new element only below some of the members of the antichain P, decide whether every embedding of P extends to an embedding of one of the Q_i. A central structural feature at the heart of this problem is the existence of an Ahmad pair: a pair of incomparable Σ^0_2 degrees a and b such that every degree c strictly below a is also below b. The renewed interest in this problem was sparked by the surprising discovery: the non-existence of an Ahmad triple. We showed that there is no triple of enumeration degrees a, b, c such that each pair a,b and b,c is an Ahmad pair.

This is joint work in progress with Goh, Lempp, and Ng.