Equivalence relations on reals, and learning for algebraic structures

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Let E and F be equivalence relations on the Cantor space 2^ω. The relation E is *computably reducible* to F (denoted by $E \leq_0 F$) if there is a total Turing functional Ψ such that for all $\alpha, \beta \in 2^\omega$,

$$(\alpha \, E \, \beta) \;\Leftrightarrow\; (\Psi^\alpha \, F \, \Psi^\beta).$$

In the talk, we discuss some recent results on $\mathbf{ER}(2^\omega)$ — the poset of $\leq_0$-degrees of equivalence relations on reals. For example, we show that the poset of Δ^0_2 equivalence relations on ω (w.r.t. a similarly defined reducibility) embeds into the interval $[\mathrm{Id}; E_0]$ inside $\mathbf{ER}(2^\omega)$. We also discuss applications of Borel equivalence relations to algorithmic learning for families of algebraic structures.

The talk is based on joint works with V. Cipriani, B. Monin, L. San Mauro, and R. Zamora.