Countable reductions, Borel equivalence relations, and computable structure theory

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The concept of reductions between equivalence relations is well established, and is often applied to equivalence relations both on ω and on Cantor space or Baire space. We begin here by investigating situations where Borel reductions between standard Borel equivalence relations on 2^ω fail to exist, and discuss the notions of *countable* and *finitary* computable reductions, which calibrate the extent to which reducibility is possible in situations in which no full computable (or even Borel) reduction exists.

We then move to ongoing joint work between Turbo Ho, Julia Knight, and the speaker, in which these ideas are applied to Turing-computable embeddings between classes of countable structures. The specific classes under discussion are the class TFAb_r of torsion-free abelian groups of (finite) rank r, and the class of all fields of transcendence degree r over $\mathbb{Q}$. For the former, it is a famous result of Hjorth and Thomas (cf. [2], for instance) that there is no Borel reduction from the isomorphism relation on TFAb_{r+1} to the same relation on TFAb_r. This fact creates a natural situation for investigating the possibilities for a countable reduction, and we present some results in this direction.

References:

[1] R. Miller, Computable reducibility for Cantor space, chapter in *Structure and Randomness in Computability and Set Theory*, eds. D. Cenzer, C. Porter, & J. Zapletal (World Scientific, 2020), 155–196.

[2] R. Miller, Computable reducibility for Cantor space, chapter in *Structure and Randomness in Computability and Set Theory*, eds. D. Cenzer, C. Porter, & J. Zapletal (World Scientific, 2020), 155–196.
[2] S. Thomas, The classification problem for torsion-free abelian groups of finite rank, *Journal of the A.M.S.* **16** (1), 233–258.